Chapter 1
Math Review Chapter Review

EQUATIONS of which to be aware: (No, knowing them won't guarantee success on your next test. You actually need to understand what each relationship *means*, when it is and is not *applicable*, and the conceptual consequences the relationship brings. In other words, as was said in the Prologue, this is a REVIEW, not a primary learning experience.)

- $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$ [In *unit vector notation*, vector $\mathbf{A}$, denoted in bold face, is equal to the vector sum of its *x component* A_x times a unit vector $\mathbf{i}$ in the *+x direction*, plus its *y component* A_y times a unit vector $\mathbf{j}$ in the *+y direction*, plus its *z component* A_z times a unit vector $\mathbf{k}$ in the *+z direction*. An example is $\mathbf{A} = -3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$.]

- $|A| = (A_x^2 + A_y^2 + A_z^2)^{1/2}$ [If a vector is defined in unit vector notation, the magnitude of the vector is equal to the square root of the sum of the squares of its components.]

- $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$ [If a vector is defined in unit vector notation, the angle between the vector and the *+x axis* is equal to the inverse tangent of the ratio of the vector's *y component* and its *x component*.]

- $\mathbf{A} = |A|\angle\theta$ [In *polar notation*, vector $\mathbf{A}$ is equal to the magnitude of $\mathbf{A}$ at an angle θ.]

- $A_x = |A|\cos\theta$ [If a vector is defined in polar notation, its *x component* can be determined by multiplying the vector's magnitude by the cosine of the angle θ, where θ is measured from the *+x axis* counterclockwise. IN SOME CASES, it is easier to generate your own right triangle and use whatever trig function is appropriate to determine that component, especially when you are dealing with vectors in the second and third quadrants.]

- $A_y = |A|\sin\theta$ [If a vector is defined in polar notation, its *y component* can be determined by multiplying the vector's magnitude by the sine of the angle θ, where θ is measured from the *+x axis*. Again, in some cases it is easier to generate your own right triangle and use whatever trig function is appropriate to determine that component.]

- $\mathbf{A} \bullet \mathbf{B} = |A||B|\cos\phi$ [If two vectors are defined in polar notation, the *dot product* between the two is defined as the product of the magnitudes of the vectors times the cosine of the angle between the vectors. If *the angle between the two vectors* isn't obvious (that is, if the two aren't drawn so as to produce a common vertex), the line of each vector must be extended until they cross. Extending each vector a little beyond that crossing exposes the appropriate angle.]

- $\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$ [If two vectors are defined in unit vector notation, the *dot product* between the two is defined as the sum of the products of the like components.]

- $|\boldsymbol{A}x\boldsymbol{B}| = |A||B|\sin\phi$ [If two vectors are defined in polar notation, the MAGNITUDE of the *cross product vector* is equal to the product of the magnitudes of the vectors times the sine of the angle between the vectors. The direction must be determined using the *right hand rule*.]

- $\boldsymbol{B}x\boldsymbol{A} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ B_x & B_y & B_z \\ A_x & A_y & A_z \end{vmatrix}$ [If two vectors are defined in unit vector notation, the *cross product* AS A VECTOR is equal to the evaluation of the three by three matrix shown. Note that the second row is made up of the components of the *first* vector denoted in the cross product. Note also that you DON'T have to use the *right hand rule* to determine the direction as the direction is determined directly through the matrix evaluation.]

COMMENTS, HINTS, and THINGS to be aware of:

- The example given above for a vector written in **unit vector notation** was ***A** = -3**i** + 4**j** - 6**k***. This is a perfectly good notation, specifically because it allows one to see whether a particular component is in the positive or negative direction. *Technically*, though, the pluses and minuses are associated with the **direction** of the vector and should be attached to the unit vector itself. That is, a technical rendering of the vector should be ***A** = 3(-**i**) + 4**j** + 6(-**k**)*. NOBODY WRITES VECTORS THIS WAY. Nevertheless, there will be times when knowing this picky little point will come in handy.

- When dealing with **polar notation**:
  --The ***magnitude*** (ALWAYS POSITIVE) is presented first.
  --The ***angle*** takes care of the sign. That is, if the vector is in the *-x direction*, you don't put a negative sign in front of the magnitude. Instead, you make the angle 180^o.
  --**When converting** from *unit vector notation* to *polar notation*, remember that your calculator will give you first and fourth quadrant angles ONLY. When you take the inverse tangent to determine an angle, make a little graph to the side to confirm which quadrant your vector should be in. If the vector turns out to be in the second or third quadrant, you will have to mess with the angle your calculator gave you to get the correct angle for the situation.

- When dealing with **unit vector notation**:
  --Don't be bashful about **creating your own right triangle**, then determining the vector components relative to the known angle in that triangle. IF YOU DO, remember that you will have to manually place negative signs into the component representations when appropriate.

- A **dot product** is a *scalar*. That is, you *can't* do a dot product between two vectors, then dot that result into a third vector. Also, dot products can be either positive or negative. The significance of the sign depends upon the situation in which the dot product is used. As an example, when we get to the energy chapter, the *work* done by a force acting on a body over a distance *d* is defined as ***F·d***. From that definition, *positive* work is associated with forces that put energy *into* a system whereas *negative* work is associated with forces that take energy *out of* a system. In any case, what is important to understand is that because dot products aren't vectors, the sign (plus or minus) of a dot product is *not* associated with a direction.

- A **dot product** yields a number that is equivalent to the product of *the magnitude of one vector* and *the magnitude of the parallel component of the second vector*. What is *the parallel component of the second vector*? It is defined as the component whose direction is *along the line of the first vector*.

- A **cross product** is a *vector*. That is, you can do a cross product between two vectors, then cross that result into a third vector.

- The **magnitude** of a **cross product** yields a number that is equivalent to the product of *the magnitude of one vector* and *the magnitude of the perpendicular component of the second vector*. What is *the perpendicular component of the second vector*? It is defined as the component whose direction is *perpendicular to the line of the first vector*.

- The **direction** of a **cross product** is ALWAYS *perpendicular to the plane* defined by the two vectors being operated upon.
  --For vectors denoted in *polar notation*, the direction must be determined using *the right hand rule*. This is weird, but we teachers don't get paid much, and making our students do apparently undignified contortions in this fashion is one of our few sources of amusement.
  --For vectors denoted in *unit vector notation*, the direction will become apparent in the evaluation of the matrix.

- There are two ways to execute **the right hand rule**.
  --The first way requires that the length of your *right hand* be extended along the line of the *first* vector denoted in the cross product. With the thumb extended out perpendicular to that direction in an open handed hitch hiking pose, the hand must be rotated until it can *wave* in the direction of the second vector (once the wave happens, the hand will be left in a closed handed hitch hiking position). Upon completion of the maneuver, the direction of the thumb will denote the direction of the cross product (remember, that direction must be perpendicular to the two vectors involved in the cross product operation).
  --The second way simply requires that the right thumb be directed along the line of the *first* vector denoted in the cross product, and the length of the fingers of the right hand be directed along the *second* vector denoted in the cross product. Once done, the direction of the cross product will be the same as the direction of a line drawn perpendicularly outward from the palm.